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# A COMMON FIXED POINT STRUCTURE

JAMES NELSON, JR.

ABSTRACT. Let $X$ be a set, $\mathscr{P}$ a collection of subsets of $X$, $\mathscr{F}$ a family of multifunctions o $X$ into itself, and $\mathscr{H}$ a family of single-valued functions of $X$ onto itself. The quadruple ($X$, $\mathscr{P}$, $\mathscr{F}$, $\mathscr{H}$ is called a common fixed point structure if there are a set of axioms which insure that for each $F$ in $\mathscr{F}$ and $h$ in $\mathscr{H}$ there is an $x$ in $X$ such that $h(x) = x \in F(x)$. A common fixed point structure of semi-trees is developed which overlaps the fixed point structures of Muenzenberger and Smithson and subsumes fixed point theorems of Wallace, Ward, Young, and Mohler.

1. **Introduction.** A *continuum* is a compact connected Hausdorff space. A continuum $X$ is *hereditarily unicoherent* if any two subcontinua of $X$ meet in a continuum. An *arboroid* is an arcwise connected and hereditarily unicoherent continuum. A metric arboroid is called a *dendroid.* If $X$ is locally connected and hereditarily unicoherent then $X$ is called a *tree.* A *multifunction* $F: X \rightarrow X$ is a point to set correspondence with $F(x) \neq \Phi$ for all $x$ in $X$. The multifunction $F: X \rightarrow X$ is said to be *upper semicontinuous* if for each closed set $C \subset X$ the set $F^{-1}(C) = \{x \in X \mid F(x) \cap C \neq \Phi\}$ is closed in $X$. The single-valued function $f: X \rightarrow Y$ is *monotone* if $f^{-1}(x)$ is connected for every $x$ in $Y$.

In [1] Borsuk showed that a dendroid has the fixed point property for continuous single-valued mappings. Then Wallace [6] proved that trees have the fixed point property for upper semicontinuous multifunctions which send points to continua. Also, as a corollary to the above, Wallace showed that if $f$ and $g$ are mappings of a continuum onto a tree with $f$ continuous and $g$ monotone, then $f$ and $g$ have a coincidence point. Ward [7] proved that Wallace's theorem remains true if "trees" are replaced by "dendroids".

Using Muenzenberger and Smithson's development of fixed point structures [4] as motivation, the author develops a common fixed point structure which subsumes the above results and other results of Smithson, Young, and Mohler.

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