SOLUTION OF TWO VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. The letters f, h, F and G denote functions with values in a normed complete ring. With suitable restrictions on these functions, it is established that

$$f(x) = h(x) + \int_a^x f(u)G(u, v)\left[\,_v\!\prod{}^x(1 + F)\right]$$

for $a \leq x \leq b$ if, and only if,

$$f(x) = h(x) + \int_a^x h(u)G(u, v)\left[\,_u\!\prod{}^x(1 + F + G)\right]$$

for $a \leq x \leq b$, and that

$$f(x) = h(x) + \int_a^x f(v)G(u, v)\left[\,_u\!\prod{}^x(1 + F)\right]$$

for $a \leq x \leq b$ if, and only if,

$$f(x) = h(x) + \int_a^x [h(v)\textstyle\sum_{j=1}^\infty G^j(u, v)]\left[\,_u\!\prod{}^x(1 + F)(1 - G)^{-1}\right]$$

for $a \leq x \leq b$.

In this paper, product integral techniques for the solution of certain types of Volterra integral equations are developed. These results connect closely with previous integral equation results in papers by B. W. Helton [3, 4] and J. C. Helton [7, 8, 9]. In addition, several other papers also contain product integral related techniques for the solution of Volterra integral equations. In particular, the reader is referred to papers by C. W. Bitzer [1, 2], J. V. Herod [11, 12], D. B. Hinton [13] and J. A. Reneke [16].

In the following, all functions are from R to N or $R \times R$ to N, where R denotes the set of real numbers and N denotes a ring which has a multiplicative identity element represented by 1 and a norm $| \cdot |$ with respect to which N is complete and $|1| = 1$. Lower case letters are used to denote functions from R to N, and upper case letters are used

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