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ON COPRODUCTS OF RECTANGULAR BANDS

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ABSTRACT. For rectangular bands X and Y, a characterization is given of the coproduct of X and Y in the category of rectangular bands and in the category of all bands. A characterization is also given of the coproduct of X and Y amalgamating a subband U.

1. **Introduction.** A semigroup is a *band* provided $x^2 = x$ for all x, and a band is *rectangular* provided $xz = xyz$ for all x, y, and z. A band X is rectangular if and only if X is the direct product of a left zero semigroup L and a right zero semigroup R [2, Theorem 4.1.5]. The main structure theorem for bands is that every band is a semilattice of rectangular bands [1]. More specifically, for a band B, Green's relation $\mathcal{D}$ is a congruence on B, $B/\mathcal{D}$ is a semilattice, and each $\mathcal{D}$-class is a rectangular band [2, Corollary 7.4.7].

When X_1, X_2 are bands, the coproduct (free product) of X_1 and X_2 in the category $\mathcal{B}$ of all bands is defined to be a band W and a pair of homomorphisms $i_t : X_t \to W (t = 1, 2)$ with the mapping property that for each band B and each pair of homomorphisms $\tau_t : X_t \to B (t = 1, 2)$, there exists a unique homomorphism $\theta : W \to B$ such that $\theta \circ i_t = \tau_t (t = 1, 2)$. When X_1 and X_2 are rectangular bands, the coproduct in the category of rectangular bands (rectangular band coproduct) would be defined by simply replacing band with rectangular band in the definition above.

This note will give a characterization of the band coproduct and rectangular band coproduct of two rectangular bands X and Y. The coproduct of X and Y amalgamating a subband U will also be characterized.

2. **The Rectangular Band Coproduct.**

LEMMA 2.1. *Let $X = L \times R$ and $Y = P \times Q$ be rectangular bands and let $\alpha : X \to Y$ be a homomorphism. Then $\alpha = (\alpha_1, \alpha_2)$ for some homomorphisms $\alpha_1 : L \to P$ and $\alpha_2 : Y \to Q$.*

PROOF. Let $\ell_1, \ell_2 \in L$, let $r_1, r_2 \in R$ and suppose that $\alpha(\ell_1, r_1) = (p_1, q_1)$, $\alpha(\ell_2, r_2) = (p_2, q_2)$. Now if $\ell_1 = \ell_2$, then $(p_2, q_2) =$

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