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NORM-DECREASING ISOMORPHISMS OF THE TRACE-CLASS ALGEBRAS OF H^* ALGEBRAS

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ABSTRACT. Let A_1 and A_2 be H* algebras and let $\tau(A_1)$, $\tau(A_2)$ be their trace classes. We show that an algebra isomorphism T of $\tau(A_1)$ onto $\tau(A_2)$ preserves the trace if any of the following conditions is satisfied:

(i) $A_1 = A_2 = A$, a simple H* algebra

(ii) T is an isometry on the minimal idempotents of A_1

(iii) T is norm-decreasing and $A_1 = A_2 = A$ is the direct sum of a finite number of simple H^* algebras. We also show that if T does preserve the trace, and it is norm-decreasing; then, the induced isomorphism T^m of the multiplier algebra $(\tau(A_1))^m$ onto $(\tau(A_2))^m$ is an isometry.

1. **Introduction.** Wendel in [10] and [11], Rigelhof in [6] and Wood in [12] and [13] have all shown that norm-decreasing isomorphism of some group algebra onto another of the same kind implies an isometry. We shall attempt to show, in this paper, that a norm-decreasing algebra isomorphism T of the trace-class algebra $\tau(A_1)$ onto another $\tau(A_2)$ which preserves the trace, induces an isometric algebra isomorphism T^m of the multiplier algebra $(\tau(A_1))^m$ onto $(\tau(A_2))^m$. The major contribution in this paper, is our investigation of when an algebra isomorphism T preserves the trace and lemma 3.1 is the basis of this investigation. The theory of the trace-class algebra itself was developed in [7] and [8]. [7] was a generalisation of Schatten's work on the trace-class algebra of operators on a Hilbert space in [9].

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2. **Preliminaries.** The trace-class for A, (denoted by $\tau(A)$) is defined to be the set $\{xy : x, y \in A\}$ (see [7]). It is dense in A by lemma 2.7 of [1]. A projection in A is a non-zero member e of A such that $e^2 = e = e^* \neq 0$ (e is a non-zero self-adjoint idempotent). We refer to a mutually orthogonal maximal family $(e_\alpha)_{\alpha \in \Gamma}$ as a projection base.

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