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APPLICATIONS OF THE INVERSE SCATTERING TRANSFORM II: THE THREE-WAVE RESONANT INTERACTION*

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ABSTRACT. The techniques in the preceding paper are applied to the three-wave resonant interaction. With the aid of computer simulations and these techniques, we can almost completely describe how this system evolves in terms of the nonlinear concepts of IST. As in the McCall-Hahn area theorem, the final areas are found to be functions of only the initial areas. Also, the final radiation densities are functions of only the initial radiation densities, and the final soliton spectrum is dependent only on the initial soliton spectrum. We discuss all three subcases and give examples of each.

1. **Introduction.** As we have already seen in the case of self-induced transparency [1], three pieces of direct scattering data determine a relatively large amount of information about the system. When the envelope of the field is real, a reflection coefficient at $\zeta = 0$ determines the area of the envelope. The energy of the envelope can be obtained from a reflection coefficient for real ζ. The bound state eigenvalues and their normalization coefficients determine the final soliton configuration.

As we turn to the three-wave resonant interaction (3WRI), we shall find that a relatively large amount of information can be obtained from the same three pieces of data. Furthermore, when this information is coupled with computer simulations of 3WRI [2], we obtain a virtually complete picture of this interaction. We should emphasize that not only will the simulations verify the theoretical predictions, but they will also give results that theory cannot obtain, as well as suggest additional theoretical interpretations.

In a unitless form, the equations for the envelopes in the 3WRI are given by

$$Q_{1t} + c_1 Q_{1x} = \gamma_1 Q_2{}^* Q_3{}^*, \tag{1a}$$

$$Q_{2t} + c_2 Q_{2x} = \gamma_2 Q_1{}^* Q_3{}^*, \tag{2a}$$

$$Q_{3t} + c_3 Q_{3x} = \gamma_3 Q_1{}^* Q_2{}^*, \tag{3a}$$

where $Q_1(x, t)$ are the envelopes, c_i are the corresponding group velocities, which are ordered according to

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