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THE DISAPPEARANCE OF SOLITARY TRAVELLING WAVE SOLUTIONS OF A MODEL OF THE BELOUSOV-ZHABOTINSKII REACTION

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ABSTRACT. We investigate a system of non-linear partial differential equations which describe spatial structure in the Belousov-Zhabotinskii chemical reaction. For various concentrations of reactants the Belousov-Zhabotinskii reaction exhibits periodic travelling waves of chemical activity, solitary travelling waves, or no waves whatsoever. It has been previously proven that there are ranges of values of f, the stoichiometric factor, over which the system has a solitary travelling wave solution and periodic travelling wave solutions. We show here that over still another range of values of f, the system cannot have a travelling wave solution.

1. **Introduction.** The Belousov-Zhabotinskii reaction is the only known chemical reaction which exhibits both temporal oscillations and spatial structure. This system is the metal ion catalyzed oxidation by Bromate ion (BrO_3^-) of easily brominated organic materials.

Temporal oscillations, first reported by Belousov [1], occur in the ratio of [Ce(IV)]/[Ce(III)]. A redox indicator such as Ferroin is often used to make the oscillations visible as sharp color changes. The periods of the oscillations may vary from seconds to minutes and they can persist for several hours since each cycle of the catalyst consumes very little of the principal reactants.

In 1970 Zaikin and Zhabotinskii reported the existence of travelling waves of chemical activity in a two dimensional system consisting of reagent spread in a thin layer over a flat surface such as a petri dish [9].

Winfree [7] showed experimentally that the two dimensional waves are of two general types. In the first case the reagent is oscillatory in time and the waves result from continuous phase gradients. These waves, called phase waves, are diffusion independent and appear to pass through impermeable barriers. The second variety of wave, called trigger waves, appear to be propagated by a reaction-diffusion mechanism. The reagent need not exhibit temporal oscillations for trigger waves to appear, and they are most striking when observed in such a solution. For example, as reported by Winfree [8],

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