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INDUCED REPRESENTATIONS OF GROUPS ON BANACH SPACES

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ABSTRACT. Let H be a closed subgroup of the locally compact group G and L a representation of H on the Banach space E. The notion of induced Banach representation is extended to the case where there exists an "inducing pair" (p, q) for L up to G. In particular, if L is bounded, then (p, p) is such a pair for any p in $[1, \infty]$. We construct an isometric representation U (depending on L, p, q) of G and resolve certain questions pertaining to this induction. We also consider some special cases of particular interest. Finally, we extend the Theorem on Induction-in-Stages and a version of the Frobenius Reciprocity Theorem to the context of inducing pairs.

Introduction. Let G be a locally compact group, H a closed subgroup of G and L a unitary representation of H on the Hilbert space E. In [13] G. W. Mackey constructed an induced unitary representation $U(L)$ of (second countable) G on a certain Hilbert space of functions. Later, R. J. Blattner [1] gave an equivalent construction for arbitrary G. It is natural to try to extend this notion of induced representation to the case where E is a Banach space or just a linear (topological) space and the operators $L(t)$, $t \in H$, are continuous and vary in a suitably smooth fashion. There are some good specific reasons for trying to do this. (1) It is well-known that the process of analytic continuation of Lie group representations forces one to consider Banach space representations and even linear system representations [8]. (2) It is also well-known that it is possible for an "induced" representation to be unitary while the original one L is not. Actually, what happens is that one constructs a bounded representation on a Hilbert space using an induction-like process and starting with a certain (generally unbounded) representation. The Hilbert space is then renormed to yield a unitary representation. This is how one obtains the so-called "complementary series" representations of semi-simple Lie groups (see [12]). Thus, the study of unitary representations itself forces one to consider induction for representations which need not be bounded. (3) In [15] C. C. Moore obtained a

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