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SWEEPING MEASURES FROM THE POLYDISC TO THE TORUS

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Let $A(\Delta^n)$ be the polydisc algebra consisting of functions continuous on the closed polydisc, Δ^n, and analytic on the interior. The distinguished boundary of the polydisc is the n-dimensional torus, T^n. This is a compact connected Abelian group. Its dual is Z^n, the cross product of n copies of the integers. Let $Z^n{}_+$ be the set of all $\alpha \in Z^n$ with $\alpha_i \in Z_+$ for $1 \leqq i \leqq n$. Let $A(T^n)$ be the algebra of continuous functions on T^n whose Fourier series vanish off $Z^n{}_+$. Theorem 2.2.1 in *Function Theory in Polydiscs* by Walter Rudin [4] shows that $A(\Delta^n)$ and $A(T^n)$ are in one-to-one correspondence. The element in $A(T^n)$ corresponding to $f \in A(\Delta^n)$ is denoted by f^*.

Let μ be a measure on Δ^n. The measure, μ, can be considered as a linear functional on $A(\Delta^n)$ and hence on $A(T^n)$. The linear functional can be extended to a linear functional on $C(T^n)$. This linear functional gives a measure, σ, defined on T^n such that

$$\int_{\Delta^n} f d\mu = \int_{T^n} f^* \, d\sigma$$

for every $f \in A(\Delta^n)$. The theorem below gives a construction method of finding the measure σ. In one variable a similar construction was given by Rubel and Shields [3].

THEOREM. *Let μ be a measure defined on Δ^n then a measure σ* can be constructed on T^n such that $\int_{\Delta^n} f d\mu = \int_{T^n} f^* d\sigma$ *for all* $f \in A(\Delta^n)$.

PROOF. Let μ be a measure on Δ^n. Then $\mu = \mu_T + \mu_0 + \mu_I$, where μ_T is the restriction of μ to T^n, μ_0 is the restriction of μ to the open polydisc, U^n and μ_I is the restriction of μ to the indistinguished boundary.

Consider first μ_0. Look at

$$\int_{U^n} f(z) \, d\mu_0(z)$$
$$= \int_{U^n} \int_{T^n} f^*(t) P(r, \theta - t) \, dm(t) \, d\mu_0(r, \theta)$$

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