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FORMS OF SINGULAR ASYMPTOTIC EXPANSIONS IN LAYER-TYPE PROBLEMS

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1. **Introduction.** A crucial part of singular perturbation techniques is to find a correct form of the asymptotic expansions used. This form should neither be so general that it precludes an actual construction of a solution nor too restrictive for the problem studied. Other aspects of techniques such as matching are closely tied up with the construction of a correct form. In this paper we shall describe how more sophisticated techniques have developed from what today (1975) seem simple techniques, and also point out some problems which need further study. By analyzing the ideas underlying techniques used in solutions of a class of problem and finding their shortcomings one may extend these techniques so as to make them applicable to more difficult problems. The progressive development of techniques will be illustrated by various examples in this paper. We shall restrict ourselves to layer-type techniques (as defined in Lagerstrom-Casten) and for simplicity we shall mainly deal with comparatively simple second-order differential equations with one small parameter. Other cases will be briefly mentioned, but not discussed, in the last section. In the problems discussed here there will be an inner and an outer expansion. From these it is desirable to construct expansions which are uniformly valid over the entire interval considered. It should be emphasized that most of the equations considered are model examples designed to give simple illustrations of techniques used in solving actual physical problems. The introduction of singular-perturbation techniques have practically always come from applied mathematicians dealing with concrete physical problems.

If ϵ is the small parameter and x is the outer variable, one has to distinguish several functions of ϵ. First the stretching parameter used for the inner layer. It may be ϵ itself so that the inner variable is $x^* = x/\epsilon$. One has, however, also to consider the possibility of coordinate changes $y = f_1(x, \epsilon)$, $y^* = f_2(x, \epsilon)$, where y is of the order of x, and y^* is of the order of x^*. Sometimes the scale of the dependent variable may also be changed. The layer of rapid transition may occur at either endpoint (boundary layer) or in the interior (for which the name shock layer, or shock structure, is used). Thus the inner variable may be of the form, say, $(x - x_0)/\epsilon$ where x_0 is a constant. Secondly one has to find suitable expansion parameters for the inner and outer expansion.

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