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A CATEGORICAL APPROACH TO THE CLOSED GRAPH THEOREM

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0. **Introduction.** The purpose of this note is to explore once again, this time from a "natural" or categorical point of view, the class of spaces which act as targets for the closed graph theorem with respect to maps from barrelled spaces. This problem has provoked over the years a series of articles in which successively broader classes of spaces are proposed as candidates for this role. Chiefly, there are the B_r-complete (= *Pták* = weakly polar) spaces of Pták [7], the slightly more general weakly *t*-polar spaces of Persson [6], and the still more general infra-*s*-spaces of Adasch [1]. The original definitions of these three classes of spaces are very similar, each being a refinement of the preceding. In the case of infra-*s*-spaces, however, a best-possible result is obtained — a characterization of precisely those spaces which are targets of the stated closed graph theorem.

Our approach to this well-known problem is categorical. Using universal constructions, we characterize what is meant for a space to satisfy the closed graph theorem with respect to barrelled spaces. This setting elucidates many previous results in the literature and simplifies their proofs. We are able to derive, for example, Adasch's characterization of infra-*s*-spaces from our criteria and some relations between infra-*s*-spaces and B_r-complete spaces. The resulting clarity and simplified proofs should, we hope, justify the approach and encourage its use elsewhere.

1. **Preliminaries.** We shall assume the terms *category* and *functor* as known. We denote the set of morphisms from A to B in the category $\mathcal{A}$ by $\mathcal{A}(A, B)$. We now state some of the basic categorical notions which are necessary in this note, a reference for which is [5].

Let $F : \mathcal{A} \to \mathcal{X}$ be a functor. The functor $U : \mathcal{X} \to \mathcal{A}$ is said to be the *right adjoint* of F (F the left adjoint of U) if for every $X \in \mathcal{X}$ there is a map in $\mathcal{X}$, $\epsilon_X : FUX \to X$, such that for every map $f \in \mathcal{X}(FA, X)$, $A \in \mathcal{A}$, there exists a unique map $\bar{f} \in \mathcal{A}(A, UX)$ such that $\epsilon_X \circ F\bar{f} = f$. The important properties of adjoints used in this note are: (1) adjoints are unique up to isomorphism; (2) a right (left) adjoint preserves inverse (direct) limits. A map $f \in \mathcal{A}(A, B)$ is an *epimorphism* if

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