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RIGHT-ORDERABLE DECK TRANSFORMATION GROUPS

F. THOMAS FARRELL

0. **Introduction.** Let $p : E \to B$ be a regular covering space such that E is path connected, and B is a Hausdorff, paracompact space with a countable fundamental group. Also let $\mathbf{R}$ denote the real line, and $q : B \times \mathbf{R} \to B$ be projection onto the first factor.

QUESTION. Does there exist an embedding $f: E \to B \times \mathbf{R}$ such that the composite of f with q is p?

We show that the answer to this question is yes, if and only if $\pi_1 B/p_{\#}\pi_1 E$ is a right-orderable group.

In addition, if B happens to be a manifold and $\pi_1 B/p_{\#}\pi_1 E$ is right orderable, then we show that $B \times \mathbf{R}$ can be foliated so that at least one of its leaves is a one-to-one continuous image of E, and the remaining leaves are one-to-one continuous images of intermediate covering spaces of B.

Rubin [10] had previously answered an important case of this Question. Namely he considered the universal cover of any space homotopically equivalent to a countable wedge of circles. Rubin's covering space result played a key role in the proof by R. D. Edwards and R. T. Miller [3] that cell-like closed-0-dimensional decompositions of R^3 are R^4 factors. Also Edwards and Miller extended Rubin's result to answer the above Question when $\pi_1 B/p_{\#}\pi_1 E$ is a countable free group.

1. **Preliminary facts about right-orderable groups.**

1.1. DEFINITION. A right-ordered group is a pair $(G, >)$ where G is a group, and $>$ is a total order on G, such that for all x, y, and z in G, $x > y$ implies that $xz > yz$. A group G is right-orderable, if there exists an order $>$ such that $(G, >)$ is a right-ordered group.

The following basic facts about right-orderable groups can be found in [1] and [4].

1.2. *Right-orderable groups are torsion-free.*

1.3. *Any free group is right-orderable. Also any free abelian group is right-orderable.*

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