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A NOTE ON SOME LEBESGUE CONSTANTS

E. W. CHENEY AND T. J. RIVLIN

Given a continuous function f on $[0, 2\pi]$ and the set of nodes

$$(1) \qquad x_j = \frac{2j+1}{2n+1}\pi, \quad j = 0, 1, 2, \cdots, 2n,$$

there exists a unique trigonometric polynomial t_n of degree at most n such that $t_n(x_j) = f(x_j)$, $j = 0, 1, 2, \cdots, 2n$. We write $L_n f = t_n$, thereby defining the interpolating projection L_n. The norm of this projection

$$(2) \qquad \lambda_n = \|L_n\| = \max\{\|L_n f\| : \|f\| \leqq 1\}$$

is called the *Lebesgue constant* of order n for trigonometric interpolation at the nodes (1). In (2) the function norms are supremum norms on $[0, 2\pi]$. It is known (cf. Morris and Cheney [3]) that

$$(3) \quad \lambda_n = \frac{1}{2n+1}\left\{1 + 2\sum_{j=1}^{n} \sec\frac{j\pi}{2n+1}\right\}$$

$$= \frac{2}{2n+1}\sum_{j=1}^{n} \csc\frac{2j-1}{2n+1}\cdot\frac{\pi}{2} + \frac{1}{2n+1}.$$

Our purpose here is to present a detailed analysis of the asymptotic behavior of λ_n. The analysis depends upon interpreting the expression in (3) as a Riemann sum for a certain integral. We apply the same technique to the classical Lebesgue constants of the Fourier series.

The main tool in the analysis is the following lemma.

LEMMA. *For any function $f \in C^3[0, 1]$ satisfying the inequalities*

(i) $f'''(x) \geqq 0, 0 \leqq r \leqq 1$, *and*

(ii) $3f'(0) + 2f''(0) \geqq 0$,

the Riemann sums

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