

CONSTRUCTING BEZOUT DOMAINS

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Introduction. Techniques for constructing Bezout domains are described and recent examples are given.

If R is an integral domain, Q the quotient field of R , and U the units of R , then $(Q - \{0\})/U$ is a partially ordered group called the divisibility group of R . The emphasis is on constructing a desirable divisibility group and then the integral domain from this ordered Abelian group. Given a totally ordered Abelian group, W. Krull used a group algebra to construct a valuation ring with that divisibility group (also known as the value group). I. Kaplansky and P. Jaffard generalized this so that given a lattice ordered Abelian group there exists an integral domain with that divisibility group. J. Ohm showed that this integral domain is a Bezout domain, and he popularized the use of this construction for generating examples. Similarly, given a totally ordered Abelian group there exists a long power series ring with that divisibility group, and this long power series ring is a maximally complete valuation ring which can be used to generate examples.

In section one the preliminaries are given. This includes the development of the Krull-Kaplansky-Jaffard-Ohm construction of Bezout domains from lattice ordered Abelian groups and the transferring of the properties between a Bezout domain and its divisibility group. In section two there is given a brief discussion of long power series rings and how they can be used to generate examples. In section three many examples of Bezout domains obtained by the Krull-Kaplansky-Jaffard-Ohm construction appear. Finally in section four related approaches to the subject are discussed, and suggestions for future study are indicated.

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