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ON THE OSCILLATION OF A CLASS OF FOURTH ORDER DIFFERENTIAL EQUATIONS

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1. **Introduction.** This paper is concerned with fourth order differential equations of the form

$$(p(x)y'')'' - q(x)y'' - r(x)y = 0, \tag{L}$$

where p, q and r are assumed to be continuous, real-valued functions on the interval $[a, \infty)$. In addition, it will be assumed throughout that $p > 0$, $q \geqq 0$ and $r \geqq 0$ on $[a, \infty)$, with r not identically zero on any subinterval. If q is a (non-negative) constant, then (L) is self-adjoint; otherwise (L) is non-self-adjoint.

The objective of the paper is to study the oscillatory behavior of the solutions of (L). A non-trivial solution y is *oscillatory* if the set of zeros of y is not bounded above. If the set of zeros of y is bounded above, which implies y has only finitely many zeros, then y is *non-oscillatory*. Hereafter, the term "solution" will be interpreted to mean non-trivial solution.

Various special cases of (L) have been studied in detail. In particular, we refer to the fundamental work of W. Leighton and Z. Nehari [**5**, Part I] on the self-adjoint equation

$$(p(x)y'')'' - r(x)y = 0. \tag{1}$$

M. Keener [**3**, Part I] continued the investigation of (1), concentrating on the oscillatory behavior of solutions. S. Hastings and A. Lazer [**2**] considered the self-adjoint equation

$$y^{(4)} - r(x)y = 0, \tag{2}$$

showing that (2) has a linearly independent pair of bounded oscillatory solutions when it is assumed that $r \in C'[a, \infty)$, with $r > 0$ and $r' \geqq 0$ on $[a, \infty)$. S. Ahmad [**1**] has also studied (2), giving necessary and sufficient conditions for the existence of a linearly independent pair of oscillatory solutions. Finally, we refer to the work of V. Pudei [**6**], [**7**] in which the equation

$$y^{(4)} - q(x)y'' - r(x)y = 0 \tag{3}$$

is considered.

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