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A DOCQUIER-GRAUERT LEMMA FOR STRONGLY PSEUDOCONVEX DOMAINS IN COMPLEX MANIFOLDS

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1. **Introduction.** Let M be a closed complex submanifold of C^n. The lemma referred to in the title is the following projection lemma of Docquier and Grauert.

1.1 THEOREM. [1] *Let K be a compact subset of M. There is a neighborhood U of K and a holomorphic map $\pi : U \rightarrow U \cap M$ such that $\pi(p) = p$ for $p \in U \cap M$.*

This lemma allows for the generalization to Stein manifolds of several function-algebraic results on domains in C^n [1, 9]. The purpose of this note is to prove a similar lemma when K is a strongly pseudo-convex domain on M, so that U is also strongly pseudoconvex, and $U \cap M = K$. We conclude with some applications, notably a generalization of the theorem of Sibony and Wermer [10].

I am indebted to Lee Stout for repeatedly encouraging me to pursue this idea, and to J. E. Fornaess for providing one critical step. Thanks also to N. Kerzman for reminding me recently of the potential usefulness of this lemma.

2. **The Proof of the Lemma.** We begin with the result of J. E. Fornaess, which can, for our purposes, be taken as the definition of s. psc. (strongly pseudoconvex).

2.1 THEOREM. [2] *Let D be a s. psc. domain on a Stein manifold M. There is a neighborhood N of $\bar{D}$, a C^∞ function ρ defined in, and s. psh. (strictly plurisubharmonic) on N such that*

(a) $d\rho \neq 0$ near ∂D,

(b) $D = \{p \in N : \rho(p) < 0\}$.

For a proof of this result see [8].

We shall call ρ a *defining function for D*. The main result of this note is the following assertion.

2.2 THEOREM. *Let M be a closed submanifold of a Stein domain U_0 in C^n. Suppose we are given (by Theorem 1.1) a neighborhood U of*

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