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ON THE WEAK CONVERGENCE OF SIMILAR PROBABILITY LAWS

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The concept of types of random variables can be extended to random vectors in several ways. It is shown in this paper that with some natural extensions of this concept, convergence of laws is in reality convergence of types as in the case of random variables.

DEFINITION 1. We say two random variables X and Y (or their probability laws) are of the same (positive) *type* if $\exists\, b > 0$ and a such that $\mathcal{L}(Y) = \mathcal{L}(a + bX)$.

DEFINITION 2. We say a random variable X (or its law) is *degenerate* if $\exists\, c$ such that $P(X = c) = 1$.

DEFINITION 3. We say a distribution function $F(x)$ is *defective* if $F(+\infty) - F(-\infty) < 1$, or equivalently, if $F(+\infty) < 1$, or $F(-\infty) > 0$ or both.

In this paper we are primarily concerned with distribution functions of random variables which are necessarily nondefective.

DEFINITION 4. We say U_n *converges weakly* to U ($U_n \xrightarrow{w} U$) or $\mathcal{L}(U_n) \to \mathcal{L}(U)$ if for every continuity point z of $F(\boldsymbol{u}) = P(\boldsymbol{U} \leqq \boldsymbol{u})$, $P(\boldsymbol{U}_n \leqq \boldsymbol{z}) = F_n(\boldsymbol{z}) \to F(\boldsymbol{z})$.

This definition holds both for random variables and random vectors.

We restate the familiar result about the weak convergence of types of random variables as follows. If, for sufficiently large n, X_n and Y_n are of the same type and $X_n \xrightarrow{w}$ nondegenerate X and $Y_n \xrightarrow{w}$ nondegenerate Y, then X and Y are of the same type [1, Theorem 1, p. 40]. It is this result that justifies the usage that a sequence of random variables Z_n has, asymptotically, a normal distribution (without specifying the norming constants). Since all (univariate) nondegenerate normal distributions are of the same type it follows that for any sequence of constants a_n and $b_n > 0$ the limit law of $a_n + b_n Z_n$ is also normal possibly with different mean and variance if it is nondefective and nondegenerate.

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