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LINEAR COMBINATIONS OF CONVEX MAPPINGS

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1. **Introduction.** Let S denote the class of functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ which are analytic and univalent in the unit disc $U : |z| < 1$. A function $f \in S$ is said to be *starlike of order* α, $(0 \leqq \alpha < 1)$, denoted $f \in S^*(\alpha)$, if

$$\operatorname{Re} \left\{ z \frac{f'(z)}{f(z)} \right\} > \alpha , \qquad (z \in U)$$

and is said to be *convex of order* α, denoted $f \in K(\alpha)$, if

$$\operatorname{Re} \left\{ 1 + z \frac{f''(z)}{f'(z)} \right\} > \alpha , \qquad (z \in U).$$

It is well known that $f \in K(\alpha)$ if and only if $zf' \in S^*(\alpha)$.

In [1, p. 38], the following question is considered: Suppose $f, g \in K(0)$. For $0 < t < 1$, set

$$h = tf + (1 - t)g. \tag{1}$$

Is h starlike and univalent?

MacGregor answered this question in the negative. In [5], he showed that h need not be univalent in any disc $|z| < r$, $r > 1/\sqrt{2}$.

In this note, we investigate functions of the form (1) when $f, g \in K(\alpha)$. For $0 < \alpha < 1/2$, a radius of univalence is found. For $\alpha = 1/2$, we show that h is univalent and close-to-convex.

2. **Radius of univalence.** The development of this section will parallel that of MacGregor in [5], with the class $K(0)$ replaced by $K(\alpha)$. For $f(z) \in K(\alpha)$, consider the related function $g(z)$ defined by $g(z) = \epsilon \overline{f(\epsilon \bar{z})}$, $|\epsilon| = 1$. Note that

$$g'(z) = \overline{f'(\epsilon \bar{z})}, \; g''(z) = \bar{\epsilon} \overline{f''(\epsilon \bar{z})}. \tag{2}$$

From (2), we obtain

$$1 + z \frac{g''(z)}{g'(z)} = 1 + z\bar{\epsilon} \frac{\overline{f''(\epsilon \bar{z})}}{\overline{f'(\epsilon \bar{z})}} ,$$

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