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ON A PAPER OF RICHMAN AND WALKER

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This note was inspired by a paper of F. Richman and E. A. Walker [5] in which, among other results, is the theorem that an abelian p-group with an unbounded basic subgroup is determined by its endomorphism ring. In fact these authors do more than give a new proof of this well-known theorem. They construct the group as a module over its endormorphism ring, not merely as Z-module. They also point out that if the group is bounded, then it is isomorphic to the left ideal of the endomorphism ring generated by any primitive idempotent of maximal additive order; thus in this case, too, constructing the group, as module, from the ring. This leaves the problem of determining the group as module over its endomorphism ring, in the case of a bounded basic subgroup but non-zero divisible subgroup. We give a solution to this problem for divisible groups in § 1, and show there too that if $G = D \oplus H$ with D divisible and H reduced, then the modules D and $R = G/D$ over $E = \mathrm{Hom}_Z(G, G)$ are determined from E. Thus, knowing E, we then know the E-modules D and R and will know the E-module G once we know the element of $\mathrm{Ext}_E{}^1(R, D)$ determined by the exact sequence $0 \to D \to G \to R \to 0$.

It turns out (§ 2) that $\mathrm{Ext}_E{}^1(R, D)$ is a cyclic module over a certain ring Γ with the class of the sequence above as a generator. Moreover two exact sequences $0 \to D \to X_i \to R \to 0$ of E-modules have isomorphic modules X_i if and only if their classes are multiples of each other by p-adic units. Finally, in § 3 it is shown that Γ is the ring of p-adic integers. In this way G is determined, as E-module, by E. Thus, in case R is bounded, we have answered the question which motivated us but no restriction on R is necessary (thanks partly to the referee — cf. § 3) so that we obtain a much more general result than we sought. Clearly, however, we require $D \neq 0$.

1. **The Modules D and R.** The object of this section is to show how the modules D and $R = G/D$ can be constructed from $E = \mathrm{Hom}_Z(G, G)$ when D is the maximal divisible subgroup of G. These results, taken

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