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ALMOST PERIODIC DINI THEOREMS

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Introduction. Dini's classical theorem [4] states that if a monotone sequence of real valued continuous functions converges pointwise to a continuous limit on a compact space, then the convergence is uniform. It is, of course, well known that the corresponding statement fails on a non-compact space. However, in light of the Bohr compactification of the real line $\boldsymbol{R}$, one might expect that the corresponding statement for Bohr almost periodic functions would be true. (See e.g., [7].) Alas, this too is false as can be seen by examples such as $f_n(t) = [(2 + \sin\sqrt{2}t + \cos t)/4]^n$, which does not converge uniformly on $\boldsymbol{R}$ even though $f_n \downarrow 0$ pointwise on $\boldsymbol{R}$ (and even though the mean values $M(f_n)$ tend to zero). Nevertheless, by adding some appropriate additional condition one does obtain true versions of a Dini-type theorem for almost periodic functions. One such version has been given by Luigi Amerio in [1] and [2]. Moreover, Salomon Bochner has given in [3] a rather general Dini-type theorem for almost automorphic functions which includes both Amerio's theorem and the classical Dini theorem (although in the latter case, as Bochner points out, his functions must be defined on a sequentially compact space rather than on a compact space). Finally Mario Dolcher [5] has given a proof of Amerio's theorem based on the Bohr compactification of the real line.

The purpose of this note is to show that Amerio's theorem can be deduced briefly from an abstract Dini theorem (Proposition 3.4 on page 91 in [10]), which was evidently first stated by Krein in [8] for partially ordered normed linear spaces, and which is already known to contain the classical Dini theorem (cf. [10] pp. 92–93). We also give another criterion, different from Amerio's, for the uniform convergence of monotone sequences of a. p. functions on $\boldsymbol{R}$.

The Abstract Dini Theorem. In order to make our argument complete and self-contained (and for the convenience of the reader) we first state and prove a simplified version of the abstract Dini theorem which on the one hand is entirely adequate for the deduction of Amerio's result and on the other hand can be given a rather short

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