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AN ALGEBRAIC TREATMENT OF ALGEBRAICALLY COMPACT GROUPS

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1. **Introduction.** Algebraically compact groups were defined by Kaplansky in [4]. It is inherent in his definition that a complete set of invariants exists for this class of groups. Kaplansky's definition was topological, and other authors, such as Fuchs [2], who have discussed the structure of these groups have taken a topological approach. In fact, it appears that no non-topological discussion of the structure theory of algebraically compact groups exists in the literature. Such a discussion was motivated as follows. The algebraically compact groups are the injectives relative to the class of pure short exact sequences. The development of the structure theory of these injectives was topological. In attempting to develop structure theories for the injectives with respect to other classes of short exact sequences, no corresponding topological methods were available on the one hand, and on the other, no purely algebraic development of algebraically compact groups seemed to exist. It was with some relief that the authors discovered that an elementary purely algebraic treatment of these groups was possible, and this paper presents that treatment.

The notation and terminology used will mainly conform to that of Fuchs [2]. We recall here a few fundamental notions. In this paper, group means Abelian group. A group G is *divisible* if $nG = G$ for all integers $n \neq 0$. Every group G has a maximum divisible subgroup dG, and dG is a summand of G. If $dG = 0$, G is called *reduced*. The injective Abelian groups are precisely the divisible ones, and injective envelopes are called *divisible hulls*. If p is a prime, then G is *p-primary*, or is a *p-group*, if every element of G has order a power of p. If G_p denotes the maximum p-primary subgroup of G, and tG denotes the torsion subgroup of G, then tG is the direct sum $\sum G_p$ of the p-groups G_p. A group G is *p-divisible* if $p^nG = G$ for all n. The group $(Q/Z)_p$ is denoted $Z(p^\infty)$, where Q is the additive group of rational numbers and Z is the group of integers. $Z(n)$ denotes the cyclic group of order n.

2. **Purity.** Before defining algebraically compact groups, we must introduce the concept of purity.

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