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DOMINANCE OF N-TH ORDER LINEAR EQUATIONS

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ABSTRACT. Consider the nth order linear equation

$$(1) \qquad y^{(n)} + \sum_{k=1}^{n} p_k(x) y^{(n-k)} = 0, \text{ where } p_k(x) \in c[a, \infty) \text{ for } 1 \leqq k \leqq n.$$

Introducing a new concept called dominance, the authors compare the asymptotic properties of the set of oscillatory solutions with the set of nonoscillatory solutions for the equation (1) when dominance occurs. These results also give information about the number of linearly independent oscillatory or nonoscillatory solutions of (1). The third order equation is given concentrated attention.

Consider the nth order linear equation

$$(1)_n \; y^{(n)} + \sum_{k=1}^{n} p_k(x) y^{(n-k)} = 0 \text{ where } p_k(x) \in c[a, \infty) \quad \text{for } 1 \leqq k \leqq n.$$

A nontrivial solution of equation $(1)_n$ is said to be oscillatory on $[a, \infty)$ if it has infinitely many zeros on $[a, \infty)$; otherwise, it is said to be nonoscillatory.

Many of the known results for nth order oscillation theory are catalogued in Swanson [1]. Also a good discussion for 3rd order equations can be found in Barrett [2].

It is the intent of this paper to compare the asymptotic properties of oscillatory and nonoscillatory solutions of equation $(1)_n$ by means of the concept of dominance. Thus if information about the asymptotic behavior of all nonoscillatory solutions is known for certain equations then asymptotic behavior of all oscillatory solutions can be determined and visa versa. This approach also yields information about the number of linearly independent nonoscillatory and linearly independent oscillatory solutions of equation $(1)_n$.

We conclude with an examination of third order equations, $(1)_3$, as examples of types of dominance.

1. **Dominance.** Let $\mathcal{S}$ denote the linear space of all solutions of $(1)_n$, $\mathcal{N}$ the subset of $\mathcal{S}$ of nonoscillatory solutions and $\mathcal{O}$ the subset of $\mathcal{S}$ of oscillatory solution. Let $\mathcal{N}^+$ be the subset of $\mathcal{N}$ of solutions

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