ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 5, Number 2, Spring 1975

THE CONSTRUCTIVE JORDAN CURVE THEOREM

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ABSTRACT. This paper presents a constructive treatment of the Jordan curve theorem. It is shown that, given a Jordan curve, and a point whose distance to the curve is positive, then there is a finite procedure to decide whether the point is inside or outside the curve. Also, given two points that are either both inside, or both outside, the curve, then there is a finite procedure that constructs a polygonal path joining the two points, that is bounded away from the curve. Finally, a finite procedure is given for constructing a point inside the curve.

1. **Introduction.** Since the publication of Bishop's book [2] on contructive analysis, there has been a resurgence of interest in the constructive approach to mathematics. The Jordan curve theorem provides a pertinent illustration of this approach. The main concern of the Jordan curve theorem is the construction of a path joining two points and missing a curve. Indeed, the heart of the theorem may be stated as follows: *Given any three points off a Jordan curve, two of them can be connected by a path missing the curve.* The constructive approach requires finding an explicit, finite procedure for computing this path.

The standard treatments of the Jordan curve theorem (see, for example, [1], [4], [5], [6]) do not address themselves to this computation, nor can they be modified easily to supply it. The usual approach is to prove abstract existence, by reductio ad absurdum, and by appeals to nonconstructive existence theorems such as the Heine-Borel theorem. The purpose of this paper is to demonstrate that such an approach is neither necessary nor desirable. By viewing the problem constructively, we are led to a proof that is as simple as any, while considerable insight is gained into a theorem which is often considered to be a triviality. For when phrased in terms of an explicit construction of a path, the difficulty becomes apparent, even if you are sure that the curve has an inside and an outside. Brouwer gave the first constructive proof of the Jordan curve theorem in a rather formidable paper [3]. Our approach parallels Brouwer's intuitionist treatment but is in the spirit of modern constructivism.

All mathematical objects dealt with here have computational meaning. A point in the plane is given by a pair of real numbers. A real number is given by providing rational numbers which approximate

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AMS (MOS) subject classifications (1970). Primary 02E05, 57A05.
Received by the editors October 2, 1972 and in revised form January 9, 1973.