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A SURVEY OF PERFECT CODES

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1. **Introduction.** In recent years there has been a lot of interest in so-called "perfect codes". Originally a topic in the theory of error-correcting codes, there are now connections to group theory, combinatorial configurations, covering problems and even diophantine number theory. As the results are spread over many fields and journals and since many of them are quite recent there has been a lot of duplication and more is to be expected. For this reason it seems worthwhile to write a survey of what is known (to this author) at the moment.

To introduce the subject and for further use in this survey we need a number of concepts from the theory of error-correcting codes which we now introduce briefly.

Consider a set F of q distinct symbols. We shall call this set the *alphabet*. By $\mathcal{R}^{(n)} := F^n$ we denote the set of all n-tuples from F, i.e.

$$(1.1)\qquad \mathcal{R}^{(n)} := \{(a_1, a_2, \cdots, a_n) \mid a_i \in F, i = 1, 2, \cdots, n\}.$$

In many cases we shall impose some algebraic structure, e.g., F will be a ring and $\mathcal{R}^{(n)}$ an F-module. If F is the field GF(q) we can consider $\mathcal{R}^{(n)}$ as an n-dimensional vector-space over F. We write $\mathbf{a} := (a_1, a_2, \cdots, a_n)$ and use the words *vector* or *word* to denote the n-tuples from $\mathcal{R}^{(n)}$ (n is called the *block length* or *word length*). We also introduce a metric d in $\mathcal{R}^{(n)}$. In coding theory the most familiar metric is *Hamming-distance* defined by

$$(1.2)\qquad \mathrm{d}(\mathbf{x}, \mathbf{y}) := \textit{the number of indices } i \textit{ for which } x_i \neq y_i.$$

In many cases one of the symbols of F is denoted by 0 (zero). We then define the *weight* $w(\mathbf{a})$ of a word $\mathbf{a}$ by

$$(1.3)\qquad w(\mathbf{a}) := \textit{the number of indices } i \textit{ for which } a_i \neq 0.$$

Note that if $\mathbf{0} := (0, 0, \cdots, 0)$ then $w(\mathbf{a}) = d(\mathbf{a}, \mathbf{0})$.

If $\mathbf{x} \in \mathcal{R}^{(n)}$ and $e \geqq 0$ is an integer we define the *sphere* $S(\mathbf{x}, e)$ with center $\mathbf{x}$ and radius e by

$$(1.4)\qquad S(\mathbf{x}, e) := \{\mathbf{y} \in \mathcal{R}^{(n)} \mid d(\mathbf{x}, \mathbf{y}) \leqq e\}.$$

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