ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 5, Number 1, Winter 1975

SEPARABLE ALGEBRAS OVER PRUFER DOMAINS

JOSEPH A. WEHLEN

ABSTRACT. Employing sheaf-theoretic techniques and idempotent lifting properties, the following extension of the Wedderburn Principal Theorem is shown: If A is any finitely generated algebra over an almost Dedekind domain R such that A modulo its Baer lower radical is separable over R, then A contains a separable subalgebra S which adds with the lower radical as R-modules to give A.

Certain structural results concerning separable algebras over arbitrary Prufer domains are obtained. These parallel results already known for Dedekind domains.

Finally, results relating the Hochschild and the weak global dimensions of an algebra with the weak global dimension of the ground ring are obtained. The ground rings include some Prufer domains and their generalizations.

The purpose of this article is to examine the structure of commutative and non-commutative algebras over Prufer domains, and in particular over almost Dedekind domains. We shall prove for these last rings a generalization of the Wedderburn Principal Theorem.

In § 1, it will be shown that every finitely generated, torsion-free, separable commutative algebra S over a Prufer domain R is again a Prufer domain. If R is an almost Dedekind domain, then so also is S.

In § 2, similar results will be obtained for non-commutative separable algebras over these same types of ground ring. Moreover, since a finitely generated, torsion-free algebra over a Prufer domain is projective, one may write any separable algebra over a Prufer domain as a direct sum of the torsion ideal and the projective ideal. This decomposition can be "lifted" to any finitely generated algebra A which is separable modulo its lower radical. An application of sheaf-theoretic techniques as in [3] to this decomposition obtains the Wedderburn-like structure theorem for almost Dedekind domains.

In § 3, we will give results relating the Hochschild dimension and the weak global dimension of a finitely generated, torsion-free algebra

Received by the editors August 9, 1973.

AMS Subject Classification (1970): Primary 13F05, 16A16, 16A46; Secondary 13E10, 16A32, 16A60.

Key words and phrases: Prufer domain, Bezout domain, almost Dedekind domain, separable algebra, weak inertial coefficient ring, inertial subalgebra, lifting idempotents, sheaves of local rings, Hochschild dimension, weak global dimension, Baer lower radical, prime radical, local rings.