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SMOOTHNESS THEOREMS FOR THE PRINCIPAL CURVATURES AND PRINCIPAL VECTORS OF A HYPERSURFACE

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1. **Introduction.** The most important invariants of a C^∞ hypersurface, M^n, immersed in R^{n+1} are its principal curvatures, the elementary symmetric functions of these principal curvatures (the so-called higher mean curvatures), and the principal vectors associated to these principal curvatures. The higher mean curvatures are clearly C^∞ everywhere on M^n, but the smoothness of the principal curvatures and the associated principal vectors is a more difficult matter. The following facts are generally known: First, the principal curvatures are continuous on all of M and differentiable on an open, dense subset of M. Second, in a neighborhood of any point in this open, dense subset, the principal vectors can be chosen to be C^∞. However, complete proofs of these facts do not seem to be available. The purpose of this paper is to give these proofs, in a somewhat more general setting than that described above. The proofs employ only elementary analysis, linear algebra, and point-set topology.

The general situation that we examine is a manifold, M^n, together with a pair of symmetric 2-covariant C^∞ tensor fields on M, one of which is positive definite. For the remainder of this paper, we denote the positive definite field by G and the other field by G'. In the case of an immersed hypersurface, these tensor fields are the first fundamental form, I, which—being a Riemannian metric—is positive definite, and the second fundamental form, II. From this pair of tensor fields, we form a $(1, 1)$ tensor field, or linear transformation field, on M; the field may be thought of as $(G)^{-1}(G')$. For an immersed hypersurface, this map is called the Weingarten map. The eigenvalues and eigenvectors of this linear transformation field are the obvious generalizations of the principal curvatures and principal vectors of a hypersurface, and it is these quantities for which we prove the smoothness theorems stated above.

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