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OPTIMAL STOPPING AND FREE BOUNDARY PROBLEMS

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Consider a random situation or a game where at each moment a gambler is to make one of two decisions: quitting or pursuing the game purely on the basis of present information; his only opponent — so to speak — is randomness. Both decisions are not equally favorable, but their efficiency depends on the unknown outcome of the game. A reward function measures this efficiency: the higher its value, the better the situation from the gambler's viewpoint. Hence he has to decide whether the future gain will outweigh the loss due to stopping or further unfavorable moves. We aim at studying the decision rules or strategies yielding the best possible average gain. The purpose of this lecture is to show the relationship between the optimal stopping problem and a free boundary value problem for the heat equation. The study of this connection has led to a deeper grasp of various natural questions about the problem and to a more qualitative description of its solutions, opposed to a discrete approach which of course would lend itself much better to numerical results.

This is not a streamlined, but rather a cursory and leisurely account on the subject; proofs and further details may be found in the author's papers [**46**, **47**]. Several excellent books and articles written on this theme have encouraged me to omit other interesting aspects. These are the recent books by H. Robbins and D. Siegmund [**37**] and A. N. Širjaev [**42**] and the articles by L. Breiman [**8**] and H. Chernoff [**11**, **12**, **13**, **14**].

1. **Introduction.** During the last decade, various authors have addressed themselves to such problems in the context of statistical decision theory, operations research and game theory. We mention here a few of these problems:

1. There is the well-known "secretary problem", which made its debut with D. V. Lindley [**30**] and received a rigorous treatment from Y. S. Chow, S. Moriguti, H. Robbins and S. M. Samuels [**15**].

Let n secretaries apply for a job. They are to be graded according to quality from the best (1) to the worst (n). There is only a single position available and the interviewer can compare the present appli-

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