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A STOCHASTIC MODEL RELATED TO THE TELEGRAPHER'S EQUATION

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We will consider a very simple stochastic model, a random walk. Unfortunately, this model is little known. It has very interesting features and leads not to a diffusion equation but to a hyperbolic one. The model first appeared in the literature in a paper by Sidney Goldstein, known to you mostly because of his work in fluid dynamics. The model had first been proposed by G. I. Taylor — I think in an abortive, or at least not very successful, attempt to treat turbulent diffusion. But the model itself proved to be very interesting.

The problem is the following: Suppose you have a lattice of points. I mean discrete, equidistant points as in Figure 1.

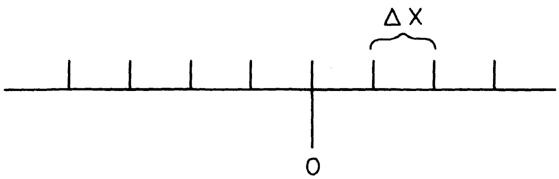

Figure 1

Now I start a particle from the original $x = 0$ and the particle always moves with speed v. It can move either in a positive direction or in the negative direction. I flip a coin, let's say, to determine which. Each step is of duration Δt and covers a distance Δx. So we have $\Delta x = v\Delta t$. Each time you arrive at a lattice point there is a probability of reversal of direction. I assume that $a\Delta t$ is to be this probability. Then, of course, $1 - a\Delta t$ is the probability that the direction of motion will be maintained.

So actually what happens is that for a time you move in the direction you have chosen. And then, all of a sudden, you flip over. For a time you move in the new direction, until again disaster overtakes you. And so you will oscillate. As is usual in such problems, what is wanted is the probability that after a certain time t the particle is at a certain interval.

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