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RECENT RESULTS ON THE STOCHASTIC ISING MODEL*

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1. **Introduction.** The stochastic Ising model was first suggested by R. Glauber [7] as a simple model for studying the time evolution of the configuration of spins in a piece of iron in a heat bath. To describe the model we let $\mathbf{Z}$ be the integers and represent a configuration of spins by a function $\eta : \mathbf{Z}^3 \to \{-1, 1\}$. The interpretation is that if $\eta(x) = 1$ (-1) the spin at x is up (down). Notice that we are approximating a piece of iron, which has a very large but finite number of spin sites, with a model that has an infinite number of spin sites. This is a standard practice called taking the infinite volume or thermodynamic limit. The infinite number of spin sites in the model causes some technical difficulties but at the same time makes the model much more interesting from a mathematical point of view.

We let each of the spins interact with its neighbors in the following way. Let $U(x, \eta)$ be given by the formula $U(x, \eta) = -\eta(x)[\sum_y \eta(y) + H]$, where the summation is over those y such that $|x - y| = 1$. $U(x, \eta)$ is to be thought of as the energy at the site x in configuration η. The parameter H is supposed to represent the external magnetic field. The idea is to have the spins at each site flipping back and forth, and the rate of flipping is to depend on the energy—high energy giving a high flip rate and low energy a low flip rate. Thus we let $c(x, \eta)$ be the rate that the spin at site x flips when the entire configuration is η and assume that $c(\cdot, \cdot)$ satisfies

(1.1) $c(x, \eta) = F(U(x, \eta))$ for some increasing function F, and

(1.2) $c(x, \eta)e^{-\beta U(x, \eta)} = c(x, {}_x\eta)e^{-\beta U(x, {}_x\eta)}$,

where $\beta > 0$ represents the reciprocal of the temperature and ${}_x\eta$ is the configuration given by

$$ {}_x\eta(y) = \begin{cases} \eta(y) \text{ if } y \neq x \\ -\eta(x) \text{ if } y = x. \end{cases} $$

One obvious choice of F is $F(z) = e^{\beta z}$ and the one which is usually used in the physics literature is $F(z) = 1/(1 + e^{-2\beta z})$. The condition (1.2) is just a technical one to guarantee that the Gibbs states (which

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