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RANDOM EVOLUTIONS: A SURVEY OF RESULTS AND PROBLEMS

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1. **Introduction and Summary.** Consider the following three physical models:

— a particle moves in a straight line with constant speed, until it suffers a random collison; then it changes velocity, and again moves in a straight line with a new constant speed.

— a radio signal propagates through a turbulent medium, in which the index of refraction is changing at random.

— a population of bacteria evolve in an environment that is subject to random fluctuations.

These are all examples of a single abstract situation, in which an evolving system changes its "mode of evolution" or "law of motion" because of random changes in the "environment" or the "medium." (In the first example, the mode of evolution is prescribed by the speed and direction of the particle; in the second, by the refractive index of the medium; and so on.)

Such situations arise in every branch of science. Recently, a general mathematical theory of such problems has been developed, the theory of "random evolutions." It is the purpose of this article to summarize the literature so far.

In physical language, a random evolution is a model for a dynamical system whose equation of state is subject to random variation. In mathematical language, a random evolution is an operator M satisfying a linear differential equation of the form

$$\frac{dM}{ds}(s, t) = -V(x(s))M(s, t) \tag{1.1}$$

or, equivalently,

$$\frac{dM}{dt}(s, t) = M(s, t)V(x(t)). \tag{1.2}$$

The coefficient V is an operator depending on a parameter x, and this parameter is stochastic. (That is, $x(t)$ is an abbreviation for $x(t, w)$, where w is a sample point in some probability space Ω.)

In this generality our model includes any homogeneous linear

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