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STIELTJES TYPE CONTINUED FRACTIONS IN QUANTUM ELECTRODYNAMICS

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ABSTRACT. The exactly solvable vacuum polarization by an external constant electromagnetic field is examined. It is proven that the Stieltjes type continued fraction corresponding to the divergent perturbation expansion in the general case does not converge to the exact solution, the convergence taking place only for the vacuum polarization for a pure electric or a pure magnetic field. The summability of the perturbation expansion to the exact solution under the Borel method is also proven.

In this communication some results will be reported concerning the divergent perturbation expansion of the vacuum polarization by an external constant electromagnetic field and its summability to the exact solution through the Stieltjes type continued fraction and the (generalized) Borel method.

The proof of the statements presented can be found in [1] and [2].

The complete Lagrangian due to the vacuum polarization by an external constant electromagnetic field, as it has been computed by Schwinger [3] reads:

$$L = -F - L_1 \tag{1}$$

$$L = -F - \frac{1}{8\pi^2}\int_0^\infty \frac{e^{-s}}{s^3}[(\mathrm{es})^2 G \frac{\mathrm{Re}\cosh(esX)}{\mathrm{Im}\cosh(esX)} - 1 - \frac{2}{3}(\mathrm{es})^2 F]\, ds \tag{2}$$

where : e is the electron charge; $F = 1/4 F_{\mu\nu}^2 = 1/2(\bar{H}^2 - \bar{E}^2)$ is the free electromagnetic field Lagrangian; $G = \bar{F} \circ \bar{H}$ is the pseudoscalar electromagnetic field invariant; $X = (2(F + iG))^{1/2}$, and the electron mass has been put equal to 1.

The first step in proving our statements is the following:

LEMMA 1. *The interaction Lagrangian* (2) *may be written under the form*:

$$L_I(\alpha) = 2\alpha^2 \int_{-\infty}^{+\infty} \frac{\psi(t)\, dt}{1 + \alpha t} \tag{3}$$

$\psi(t)$ *being positive in* $(-\infty, +\infty)$ *with finite moments given by the formula*: