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A PRIORI TRUNCATION ERROR ESTIMATES FOR CONTINUED FRACTIONS $K(1/b_n)$

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The primary goal is to obtain *a priori* truncation error estimates of continued fractions of the form

$$K(1/b_n) = \frac{1}{b_1} + \frac{1}{b_2} + \cdots,$$

where for each $n = 1, 2, \cdots, b_n \in E_n$, and the E_n are subsets of the complex plane called element regions. The method employed is based upon a correspondence between sequences of element regions and sequences of value regions which determine a nested sequence of disks. Truncation error bounds are obtained by estimating the diameter of the nth disk which contains the nth approximant $f_n = A_n/B_n$ of the continued fraction; the A_n and B_n denote the nth numerator and denominator respectively.

The element regions E_n, can be disks, half-planes, and/or complements of disks. The following theorem, from which the results of Hillam, Sweezy and Thron ([2], [3]) are easily derived, is a typical result. In this theorem the E_n are complements of disks.

Let $\{c_n\}$ be a sequence of complex numbers and let $\{r_n\}$ and $\{\delta_n\}$ be sequences of real numbers such that

$$0 \leqq |c_n| < r_n, \delta_1 = 1, 0 < \delta_n \leqq 1, n \geqq 0. \tag{1}$$

Let $K(1/b_n)$ be a continued fraction with elements b_n satisfying the conditions

$$\left| b_n + c_n + \frac{\bar{c}_{n-1}}{r_{n-1}^2 - |c_{n-1}|^2} \right| \geqq r_n + \frac{t_{n-1}}{\delta_n(r_{n-1}^2 - |c_{n-1}|^2)} . \tag{2}$$

If $f_n = A_n/B_n$ denotes the nth approximant of $K(1/b_n)$, where A_n and B_n are the nth numerator and nth denominator respectively, then for $n \geqq 2, p \geqq 0$

$$\begin{aligned} |f_{n+p} - f_n| &\leqq 2r_0 \prod_{j=2}^{n} g_j(\gamma_{j-1}, \delta_j) \\ &\leqq 2r_0 \prod_{j=2}^{n} M_j(\delta_j) \leqq 2r_0 \prod_{j=2}^{n} \delta_j \end{aligned} \tag{3}$$

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