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CONTINUED FRACTION SOLUTION TO FREDHOLM INTEGRAL EQUATIONS

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I. **Introduction.** Since the solutions to Fredholm integral equations are meromorphic functions, it seems desirable to develop rational approximations for these solutions. In this paper we describe an algorithm to obtain the continued fraction representation of the solution to these equations.

We record some useful properties of the solution to Fredholm's equation in Section II. In Section III we describe the algorithm for constructing continued fraction solutions to the equation.

II. **The Fredholm Equation.** We concern ourselves with the equation

$$f(x) = g(x) + \lambda \int_0^1 K(x, t) f(t)\, dt, \tag{2.1}$$

in which it is assumed that $K(x, t)$ is continuous on its domain and that $g(x)$ is continuous on $[0, 1]$. The Fredholm solution and the Neumann series solution of (2.1) are, respectively,

$$f(x) = \sum_{j=0}^{\infty} r_j(x)\lambda^j \Big/ \sum_{j=0}^{\infty} s_j\lambda^j, \tag{2.2}$$

and

$$\begin{aligned} f(x) &= \sum_{j=0}^{\infty} K_j(x)\lambda^j, \\ K_0(x) &= g(x),\ K_{j+1}(x) = \int_0^1 K(x, t)K_j(t)\, dt. \end{aligned} \tag{2.3}$$

Here, (2.2) is a meromorphic function of λ and (2.3) converges for restricted values of λ.

We record an extension of a definition and a theorem from Wall [1, ch. 20]. Theorem 2 is proved in Taylor [2, p. 315]. The proof of Theorem 3 is straightforward and so is omitted. These Theorems give conditions under which the solution to (2.1) has a continued fraction expansion of a certain form.

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