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CONVERGENCE OF PADÉ APPROXIMANTS IN THE GENERAL CASE

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ABSTRACT. We shall present here a new method for proving the convergence of Padé approximants in the non-Stieltjes case. This method allows us to derive results which generalize Nuttall's theorem [2] on the convergence in measure of Padé approximants for meromorphic functions. We can prove convergence in measure for a class of functions meromorphic in a circle. We show that the same kind of results can be established for generalized Padé approximants, constructed from equations given by their values at certain points instead of their Taylor series.

I. **Outline of the results.** The convergence of $[N \pm j, N]$ Padé approximants has been known for a very long time in the case of Stieltjes functions [1]. Very precise results have been obtained in this case (See G. Baker review talk at this Conference). Little was known about the general case until recent years. The reason for this seems to be that convergence in the ordinary sense is not to be expected in general. The property which plays the essential role in the proof of convergence in the Stieltjes case is that the zeros of the denominators of Padé approximants can easily be located. In the general case, it is very likely that the poles of Padé approximants can even become dense in the complex plane. So probably only weaker forms of convergence can be shown. A result of this kind has been presented at this Conference by Nuttall [2]:

Let $f(z)$ be any given analytic function, meromorphic in the complex plane. Let R, ϵ, δ be three arbitrary positive numbers. It is always possible to find an integer N such that for $n \geqq N$ the $[n, n]$ Padé approximants satisfy

$$|f(z) - P_n(z)/Q_n(z)| < \epsilon, \text{ for } |z| \leqq R,$$

except on an open set D_n of measure smaller than δ. The diagonal Padé approximants converge in measure towards $f(z)$.

We shall present here a new derivation and a generalization of this result.

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[1]Talk presented at the "International Conference on Padé Approximants, Continued Fractions and related Topics" June 21, 1972, Boulder, Colorado.