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SOME RECENT DEVELOPMENTS IN THE THEORIES OF CONTINUED FRACTIONS AND THE PADÉ TABLE

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1. **Introduction and notations.** We shall describe some new results in the theories of continued fractions and the Padé table. It is assumed that the reader is familiar with the elements of the theory of continued fractions (as given, for example, in [1] and [2]) and with the definition of the Padé quotient ([1] Ch. 5, [2] Ch. 20).

Use will be made of the following notations. The symbol $r \equiv I_i^j$ is used to indicate that an accompanying statement holds for $r = i, i+1, \cdots, j$; $r \equiv I_i$ indicates that the statement holds for $r = i, i+1, \cdots$; $r \equiv I$ indicates that it holds for $r = 0, 1, \cdots$; a symbol such as $r, m \equiv I$ is used in place of $r \equiv I$, $m \equiv I$. $r \in I$ $[r \in I_1]$ means that r is a fixed finite nonnegative [positive] integer. Single summation is tacitly understood to hold with respect to the dummy variable $\nu : \sum_i^j a_\nu$ represents $\sum_{\nu=i}^j a_\nu$; furthermore, $\sum_i a_\nu$ and $\sum a_\nu$ represent $\sum_{\nu=i}^{\infty} a_\nu$ and $\sum_{\nu=0}^{\infty} a_\nu$ respectively. Double summation is tacitly understood to hold with respect to the dummy variables ν and ν'; thus $\sum_i^j \sum_0^\nu a_{\nu,\nu'}$ represents $\sum_{\nu=i}^j \sum_{\nu'=0}^\nu a_{\nu,\nu'}$. Products are formed with respect to the variable $\tau : \prod_i^j a_\tau$ represents $\prod_{\tau=i}^j a_\tau$. $\mathfrak{D}_\mu$ denotes differentiation with respect to μ; thus $\mathfrak{D}_\mu{}^2 f(\mu)$ represents $d^2 f(\mu)/d\mu^2$. The continued fraction

$$b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2 +} \cdots \frac{a_\nu}{b_\nu} + \cdots$$

is represented by the symbol $\{b_0 + ; a_\nu : b_\nu + \}$; if b_0 is either missing or has the value zero, the symbol $\{a_\nu : b_\nu + \}$ is used; if, as sometimes occurs, a_1 and b_1 are not given by the same law of formation as that which determines the remaining $\{a_\nu\}$ and $\{b_\nu\}$, the extended symbol $\{b_0 + ; a_1 : b_1 + ; a_\nu : b_\nu + \}$ is used. The rth convergent of a prescribed continued fraction C is denoted by $C[C]_r$; thus $C[\{a_\nu : b_\nu + \}]_0 = 0$, $C[\{a_\nu : b_\nu + \}]_1 = a_1/b_1$, and so on. The Hankel determinant [3] of order $r+1$ $(r \geqq 0)$ whose $(\tau+1)$th row consists of the numbers $f_{m+\tau}, f_{m+\tau+1}, \cdots, f_{m+\tau+r}$ $(\tau \equiv I_0^r)$ is denoted by $H[f_{\tau+m}]_r$; we set $H[f_{\tau+m}]_{-1} = 1$. If $H[f_\tau]_r \neq 0$ $(r \equiv I)$, the series $\sum f_\nu z^\nu$ generates a nonterminating associated continued fraction ([4–11], [1] Ch. 3, [2] Ch. 11) of the form $\{f_0 : 1 + w_1 z + ; v_\nu z^2 : 1 + w_\nu z + \}$;

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