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P-FRACTIONS AND THE PADÉ TABLE[1]

ARNE MAGNUS

The regular continued fraction of a positive real number x_0 is obtained by writing x_0 as the sum of the greatest integer $[x_0]$ in x_0 and a remainder r_1, $0 \leqq r_1 < 1$, that is, $x_0 = [x_0] + r_1$. If $r_1 > 0$ we replace the "small" number r_1 by the "large" one $1/r_1 = x_1$ and repeat the process with x_1, that is;

$$x_0 = [x_0] + r_1 = [x_0] + \frac{1}{1/r_1}$$

$$= [x_0] + \frac{1}{x_1} = [x_0] + \frac{1}{[x_1] + r_2}.$$

Continuing in this fashion and setting $[x_i] = b_i$, $i = 0, 1, 2, \cdots$, we arrive at the finite or infinite regular continued fraction for x_0

$$x_0 = b_0 + \frac{1}{b_1} + \frac{1}{b_2} + \cdots.$$

We follow an analogous procedure for the power series

$$f = \sum_{n=-N_0}^{\infty} a_n x^n$$

$$= a_{-N_0} x^{-N_0} + \cdots + a_{-1} x^{-1} + a_0 + a_1 x + \cdots.$$

The "small" part of f is the series $\sum_1^\infty a_n x^n$ whose first non-vanishing term we denote by $a_{N_1} x^{N_1}$ and formally write

$$(a_{N_1} x^{N_1} + a_{N_1+1} x^{N_1+1} + \cdots)(a'_{-N_1} x^{-N_1} + a'_{-N_1+1} x^{-N_1+1} + \cdots) = 1,$$

where a'_{-N_1+n} is uniquely determined by $a_{N_1}, \cdots, a_{N_1+n}$, for $n = 0, 1, 2, \cdots$. We set $\sum_{-N_0}^{0} a_n x^n = b_0$ and have

$$f = \sum_{n=-N_0}^{\infty} a_n x^n = b_0 + 1 \Big/ \sum_{n=-N_1}^{\infty} a'_n x^n.$$

The process is continued to produce a finite or infinite continued fraction, called the principal part expansion of f,

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