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ANALYSIS OF TRUNCATION ERROR OF APPROXIMATIONS BASED ON THE PADÉ TABLE AND CONTINUED FRACTIONS

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1. **Introduction.** In the study and application of continued fractions

$$f = K(a_n/b_n) = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots \tag{1}$$

it is important to have realistic estimates of the truncation error $|f - f_n|$ when (1) is approximated by its nth approximant f_n. Truncation error bounds are of two main types: (a) *A priori bounds* are expressed directly in terms of the elements a_n, b_n or parameters associated with these elements. (b) *A posteriori bounds* are generally of the form

$$|f - f_n| \leqq M_n |f_n - f_{n-1}| \tag{2}$$

and are obtained only after calculating the approximants $f_1, f_2, \cdots, f_n$. (There are also asymptotic estimates of the truncation error as given by [**4**, **15**]; however, such estimates will not be dealt with here.) Bounds of a priori type can be found in [**5**, **6**, **8**, **9**, **10**, **12**, **14**, **17** and **18**] and of a posteriori type in [**1**, **2**, **3**, **6**, **7**, **9**, **11**, **13**, **16**]. Most of the known truncation error bounds for continued fractions have been obtained either by studying inclusion regions for the approximants (Section 2) or by showing that the approximants form simple sequences (Section 3). In some cases both of these approaches have been used (Section 2). This paper provides a brief summary of the two approaches and reviews some of the main results. Proofs are omitted; however, proofs, application, and numerical examples may be found in references cited. These results have a strong connection with Padé tables. As an illustration of this connection we note that in a normal Padé table the approximants of the corresponding continued fraction of the form

$$\frac{a_0}{1} + \frac{a_1 z}{1} + \frac{a_z z}{1} + \cdots \text{ (complex } a_n \neq 0)$$

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