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MONOTONE SURJECTIONS HAVING MORE THAN ONE FIXED POINT

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1. **Introduction.** Suppose X is a continuum and f is a self-mapping of X which has a fixed point e. Under what circumstances is there another point of X, distinct from e, which is also fixed under f? Very simple considerations suffice to indicate that in order to establish any kind of satisfactory theorem, e must be an endpoint in some appropriate sense, f must be surjective, and f must be more than merely continuous.

Questions of this type were studied as long ago as 1930 by W. L. Ayres [**1**]. In 1944 the first direct antecedent of the present paper appeared in a theorem of G. E. Schweigert [**16**]. Schweigert proved that if X is a dendrite, if e is an endpoint of X (that is, e is a point of order one), and if $T(X) = X$ is a homeomorphism such that $T(e) = e$, then $T(x) = x$ for some $x \in X - \{e\}$. Soon thereafter, A. D. Wallace [17] proved this theorem in case X is *any* locally connected continuum. When one attempts to prove this result for a larger class of mappings than the homeomorphisms then it becomes clear that the continua X must be drastically restricted. The author has shown [19] that for monotone surjections on locally connected continua, the existence of a fixed endpoint e implies the existence of a "small" invariant subcontinuum not containing e. The Schweigert theorem for monotone surjections is an immediate corollary. More recently W. J. Gray [9] has studied the same class of questions for finitely generated commutative semigroups of monotone surjections. He further generalized the Schweigert theorem by showing that if such a semigroup of mappings on a dendrite has a common fixed endpoint e, then it must have a common fixed point distinct from e.

It is the primary purpose of this paper to study these questions for monotone surjections on dendroids. It is proved that if two monotone surjections have a common fixed endpoint, and if they commute, then

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