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QUASILINEAR EQUATIONS AND EQUATIONS WITH LARGE NONLINEARITIES[1]

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1. **Introduction.** Two kinds of equations studied in nonlinear functional analysis are: (i) quasilinear equations, i.e., equations which differ from a linear equation by a small nonlinear term, usually a nonlinear expression multiplied by a parameter μ which is restricted to small values and (ii) nonlinear equations with well-behaved but "large" nonlinearities. These two kinds of equations are usually treated by different methods, and different kinds of results are obtained. (More extensive and detailed results can be obtained for the quasilinear equations.) Our purpose here is to study nonlinear equations with large nonlinearities by regarding them as quasilinear equations in which the parameter μ is allowed to take "large" values, i.e., we let $\mu \in [0, 1]$. We will study the equations with large nonlinearities by obtaining results about the quasilinear systems which are valid if the parameter μ is allowed to take "large" values. We study an equation in a linear space of the form

$$L(x) + \mu T(x, \mu) = 0, \tag{E}$$

where L is linear and T is a compact transformation which satisfies a uniform Lipschitz condition and a condition on the rate of growth of $\|T(x, \mu)\|$ as $\|x\|$ increases. In this paper, we consider only the case in which L has an inverse. (In a later paper, a different hypothesis will be used). By using Brouwer degree or Leray-Schauder degree theory, we obtain a theorem concerning solutions of equation (E). This theorem is then applied to obtain results on the existence of periodic solutions of ordinary differential equations, periodic solutions of functional differential equations, solutions of the Dirichlet problem for nonlinear elliptic equations and solutions of the Dirichlet problem for nonlinear parabolic equations.

In Section 2, we obtain the abstract theorem. In Section 3, we apply the theorem to obtain periodic solutions for nonlinear systems of

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