# ON SOME OLD PROBLEMS OF FIXED POINT THEORY

## ROBERT F. BROWN

It is not uncommon to begin an expository paper with the modest admission that the paper contains no new mathematics. I must make the even more modest admission that much of what I will write over-laps other expositions; especially the excellent paper of Fadell [17]. The purpose of this paper, however, is quite different from Fadell's. He described some of the new discoveries in fixed point theory; I wish to call attention to some of the problems which are, nevertheless, still with us.

1. **Continua in the Plane.** Let $X$ be a space and $f: X \to X$ a map (continuous function), then $x \in X$ is a *fixed point* of $f$ if $f(x) = x$. A space $X$ has the *fixed point property* [*for homeomorphisms*] if every map [homeomorphism] $f: X \to X$ has a fixed point. A *continuum* is a compact connected Hausdorff space. Denote the plane by $R^2$.

PROBLEM 1. *If $X \subset R^2$ is a continuum such that $R^2 - X$ is connected, does $X$ have the fixed point property?*

I do not know who first asked the question, but its age can be estimated from the reference to it in 1929* as a "well-known problem" [1]. In order to discuss the history of this problem, we must introduce its little brother:

PROBLEM 1½. *Under the hypotheses of Problem 1, does $X$ have the fixed point property for homeomorphisms?*

The first solution to Problem 1½ was by Ayers [1] (1929), but under the additional hypothesis that $X$ be locally connected. It was soon

*The date of publication of a journal article can be very misleading information when one is trying to understand the history of a mathematical problem because there is so much variation in the length of time it takes for a paper to be published after it is submitted to a journal. Therefore, the dates used in this paper are always the earliest that can be associated with the referenced paper. Whenever possible, we have used the date the manuscript was received by the editors, or the date of presentation to a scholarly meeting, in preference to the publication date.