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COVERING COMPLEXES WITH APPLICATIONS TO ALGEBRA

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The Schreier-Nielsen theorem states that every subgroup of a free group is free. There are many proofs of this, but the most perspicacious, in my opinion, is the "topological" proof of Baer and Levi [**2**], which exploits the relationship between a topological space X and the interplay between the subgroups of its fundamental group and certain "covering spaces" $\tilde{X}$ of X. It is well known (for example, see [**6**] or [**16**]) that if we concentrate on the space X and its covering spaces $\tilde{X}$, various properties enjoyed by X are inherited by each of its covering spaces $\tilde{X}$. In particular, if X is a polyhedron, so is $\tilde{X}$. Since a polyhedron can be described without topology (as an *abstract simplicial complex*), and since an analog of the fundamental group (the *edge path group*) can be defined for abstract simplicial complexes, it has long been known that the usual interplay between spaces and fundamental groups can be described abstractly, without topology. In principle, then, there is nothing new in the exposition given here, so it is a reasonable question why this article should be written.

Before answering this question, let us remark that the interplay between covering spaces and fundamental groups is a Galois (rather a "co-Galois") correspondence; there thus appears to be a second inroad of this theory into algebra. Finally, from the other side, algebraists have been well aware of all these facts and, using certain graph-theoretical constructions, "Cayley diagrams", have given proofs of the Nielsen-Schreier subgroup theorem and other more difficult theorems of Kuroš and Gruško [5; 14; 18; 19]. See also Serre's notes [**15**] on "tree products" of groups. However, the algebraists have not been successful in abstracting the topological theory in such a way that it simultaneously gives the subgroup theorems and Galois theory. This is the program here. The only new idea appears to be the definition of "covering complex" (see below) which is merely a slavish imitation of the usual topological definition. There are three sections: 1. Complexes and Edgepath Groups; 2. Covering Complexes and Subgroup Theorems; 3. Galois Theory.

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