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# STRONGLY RIGID RELATIONS

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ABSTRACT. Vopěnka, Pultr and Hedrlín proved in 1965 that on any set $A$ there exists a binary rigid relation $\rho$, i.e. a relation such that the identity transformation is the single homomorphism (compatible mapping) of $\rho$ into $\rho$. We prove the existence of a strongly rigid binary relation on any set with at least three elements. It is a relation such that all homomorphisms of $\rho^n$ into $\rho$ are projections for all $n = 1, 2, \cdots$. We characterize all strongly rigid relations on a set with two elements. Our result can be also stated as follows: There exists a binary (if $|A| > 2$) or ternary (if $|A| = 2$) relation $\rho$ on $A$ such that the trivial universal algebra $\langle A; \Phi \rangle$ is equivalent to $\langle A; A_\rho \rangle$ where $A_\rho$ is the set of all operations on $A$ preserving $\rho$.

1. Let $A$ and $I$ be sets such that $|A| > 1$, $|I| > 0$. Let $A^I$ be the set of all mappings from $I$ to $A$. Any subset $\rho$ of $A^I$ will be called an *$I$-relation* or *$|I|$-ary relation* on A. If $|I| = k < \aleph_0$ we will identify $A^I$ with $A^k$ and, in particular, for $|I| = 1, 2, 3$ any $I$-relation is simply a unary, binary or ternary relation on $A$. Let $\rho_i$ be $I$-relations on $A_i$ $(i = 1, 2)$. A mapping $f : A_1 \to A_2$ is a *homomorphism* of $\rho_1$ into $\rho_2$ (or $\rho_1\rho_2$ *compatible mapping* [13]) if $g \in \rho_1$ implies $f \circ g \in \rho_2$. A homomorphism $f : A \to A$ of $\rho$ into $\rho$ is called an *endomorphism*. A relation $\rho$ is *rigid* [13] if the identity transformation is the single endomorphism of $\rho$. The existence of a binary rigid relation on any set is proved in [13].

Given an $I$-relation $\rho$ on $A$ and $0 < n < \aleph_0$ we define the $I$-relation $\rho^n$ on $A^n$ as follows: $f \in \rho^n$ if there exist $f_i \in \rho$ $(i = 1, \cdots, n)$ such that $fx = \langle f_1x, \cdots, f_nx \rangle$ for all $x \in I$. For $1 \leqq i \leqq n < \aleph_0$ define the projections [4] (called sometimes *selective* or *trivial operations*) $e_i^n : A^n \to A$ by $e_i^n x_1 \cdots x_n = x_i$ for all $x_1, \cdots, x_n \in A$. Finally set $J = \{e_i^n \mid 1 \leqq i \leqq n < \aleph_0\}$.

DEFINITION. *Let $\rho$ be an $I$-relation on $A$. The set of all homomorphisms of $\rho^n$ into $\rho$ $(1 \leqq n < \aleph_0)$ will be denoted by $A_\rho$. The relation $\rho$ will be called a strongly rigid relation if $A_\rho = J$.*

The sets $A_\rho$ were introduced in [3] for $|I| \leqq |A| < \aleph_0$ and used in [1], [2], [14] and [7] – [12]. Obviously $f \in A_\rho$ if and only if $\rho$ is a subalgebra of $\langle A^I, \{f\} \rangle$. A relation $\rho$ is strongly rigid if and

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