ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 3, Number 3, Summer 1973

POINCARÉ DUALITY AND POSTNIKOV FACTORS

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With the advent of generalized homology and cohomology theories during the last decade there has been a natural movement toward recasting classical results from ordinary homology into these new theories. One of the earliest of the generalizations was the Poincaré duality theorem [**10**]. Perhaps the most interesting problem in this area has been the characterization of the class of orientable manifolds corresponding to each new theory ([**6**], [**7**], [**9**] and [**10**]). For example a manifold is orientable for stable homotopy if it is stably parallelizable, for real K-theory if it admits a Spin structure and for complex K-theory if it admits a Spin^c structure.

In [**7**] Kan and Whitehead asked if it is possible to fill the gap between the Eilenberg-Mac Lane spectrum $K(Z)$ (integral homology) and the sphere spectrum **S** (stable homotopy) with interesting spectra and corresponding classes of orientable manifolds. Using semi-simplicial methods, they constructed a sequence of ring spectra for which orientability was characterized by the vanishing of certain higher order ordinary homology operations on the integral fundamental class.

Our purpose here is to give an alternate, but related, solution to this problem and in the process to present an approach which should shed light on the orientability question in general. The basic tool is the Atiyah-Hirzebruch-Dold spectral sequence ([**2**]—[**5**]). Starting with an axiomatic slant product, it is shown that a corresponding product is induced on the associated spectral sequences which satisfies a derivation property with respect to the differentials (§2). This allows us to proceed from ordinary Poincaré duality at the E^2-stage to prove the generalized theorem at the E^∞-stage. A natural result of this approach is that the Poincaré duality isomorphism is seen to be an isomorphism of filtered groups, under the natural CW filtrations (§3).

This leads to the characterization of orientability as the requirement that the fundamental class at the E^2-stage be a permanent cycle. Then one might ask if there are intermediate homology theories for which a manifold is orientable if and only if its ordinary fundamental class remains a cycle until the E^k-stage. This question is answered

Received by the editors August 10, 1971 and, in revised form, December 6, 1971.
AMS (MOS) subject classifications (1970). Primary 55B20, 55B45, 55C05, 55H25; Secondary 57B10, 55J30.