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APPLICATIONS OF GLOBAL ANALYSIS TO SPECIFIC NONLINEAR EIGENVALUE PROBLEMS[1]

M. S. BERGER

Introduction. The aim of the following four lectures is threefold in that we wish to point out:

First, just how nonlinear eigenvalue problems arise in certain specific mathematical areas and to what extent the existing theory of nonlinear eigenvalue problems is successful in treating these specific applications.

Secondly, new results in the theory of nonlinear eigenvalue problems can be obtained by means of a careful study of specific mathematical disciplines.

Finally, we shall indicate basic research areas in the theory of nonlinear eigenvalue problems which are suggested by the above special problems.

At this point some general remarks and definitions are in order. Indeed, what is meant by the term "*nonlinear eigenvalue problem*"? In these lectures this term will mean the problem of studying all the solutions of certain operator equations $F(x, \lambda) = 0$, particularly in their dependence on the parameter λ. Here $F(x, \lambda)$ is an operator (generally nonlinear) defined on an open subset U of the Banach space $X \times Z$ with range in the Banach space Y. The parameter space Z will generally be R^1 or possibly R^N. Thus if $F(x, \lambda) = \lambda I - L$, and L is a bounded linear operator of a Banach space X into itself, we are concerned with the spectral theory of L. It is the merit of the nonlinear eigenvalue problems considered here that the *totality* of solutions of a given operator equation is given prime consideration. This is crucial when the operator F depends nonlinearly on x. It will be convenient to divide nonlinear eigenvalue problems into 4 parts: (i) bifurcation theory (the study of the solutions of the equation $F(x, \lambda) = 0$ near a point (x_0, λ_0) at which $F(x_0, \lambda_0) = 0$ and the Fréchet derivative of $F(x, \lambda)$ with respect to x at (x_0, λ_0), $F'(x_0, \lambda_0)$, has a nontrivial kernel); (ii) global theory (the study of solutions (x, λ) of $F(x, \lambda) = 0$ without regard to the norm of (x, λ) or the existence of nearby approximations (x_0, λ_0) for (x, λ)); (iii) singular perturbation theory (the study of the behavior of solutions of $F(x, \lambda) = 0$ as $\lambda \to \infty$); (iv) continuation theory (the study of the relations

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