

## INVARIANT MEANS ON ALMOST PERIODIC FUNCTIONS AND FIXED POINT PROPERTIES

ANTHONY TO-MING LAU<sup>1</sup>

1. **Introduction.** Consider on a topological semigroup  $S$  the following fixed point properties:

(F) For any separately continuous, equicontinuous and affine action of  $S$  on a compact convex subset  $K$  of a separated locally convex space,  $K$  has a common fixed point for  $S$ .

(G) For any separately continuous and nonexpansive action of  $S$  on a compact subset  $K$  of a separated locally convex space,  $K$  has a common fixed point for  $S$ .

Recently, Holmes and the author have proved in [10, Corollary 1] that if  $S$  is *left reversible* (i.e., any two nonempty closed right ideals of  $S$  have nonvoid intersection; see [1, p. 34]), then  $S$  has property (G). For discrete left reversible semigroups, this latter result is due to T. Mitchell [14]; the implication was first proved by De Marr in [6, p. 1139] for commuting semigroups and then by W. Takahashi [16, p. 384] for discrete left amenable semigroups (i.e., the space of bounded real valued functions on the semigroup has a left invariant mean; see Day [2]).

A well-known theorem of Kakutani [7, p. 457] shows that if  $S$  is a group, then  $S$  has property (F). This result has also been generalised recently by Šneperman [19] and [20] to the class of left reversible discrete semigroups.

Note that, as known, any commuting semigroup is left amenable (see Day [2, p. 516]) and any left amenable discrete semigroup is left reversible (see Granirer [8, p. 371]).

The main purpose of this paper is to show that, for any topological semigroup  $S$ , the existence of a left invariant mean on  $AP(S)$ , the space of strongly almost periodic functions on  $S$ , is equivalent to *each* of the two fixed point properties (F) and (G).

Since if  $S$  is left reversible then  $AP(S)$  has a left invariant mean (note that the converse is false; see [10, §4]), it follows that our result generalises Šneperman's fixed point theorem in [7, p. 457] and a fixed

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