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ON THE SIGN OF THE GREEN'S FUNCTION BEYOND THE INTERVAL OF DISCONJUGACY[1]

A. C. PETERSON

1. **Introduction.** We are concerned with the nth order quasi differential equation

$$(\mathrm{E}_n) \qquad L_n[y] \equiv (D_{n-1}y)' + \sum_{i=1}^{n-1} f_{ni}D_{i-1}y = 0$$

where the quasi derivatives are given by

$$D_0 y = y, \qquad D_1 y = f_{12}^{-1} y',$$

$$D_i y = \frac{1}{f_{i,i+1}} \left[(D_{i-1}y)' + \sum_{j=1}^{i-1} f_{ij}D_{j-1}y \right],$$

$i = 2, \cdots, n-1$. We assume that the functions $f_{ij}(x)$, $i, j = 1, \cdots, n$, are continuous on $(-\infty, \infty)$, $f_{ij}(x) \equiv 0$, if $i + j$ is even or $j > i + 1$, $f_{i,i+1}(x) > 0$ on $(-\infty, \infty)$ for $i = 1, \cdots, n-1$. This nth order canonical form (Zettl [**1**]) was introduced by J. H. Barrett [**2**] for $n = 3$ and $n = 4$. If $[\alpha, \beta]$ is an interval of disconjugacy, the sign of the Green's function for the multi-point boundary value problems of de la Vallée Poussin is well known [**3**]. For the classical third order linear differential equation, the Green's function for either of the two point boundary value problems of de la Vallée Poussin conserves its sign if and only if $[\alpha, \beta]$ is an interval of disconjugacy [**4**]. The main result of Aliev [**5**, Theorem 4] is to show that, for the classical fourth order linear differential equation, it is not necessary that $[\alpha, \beta]$ be an interval of disconjugacy in order for the Green's function for either the (3, 1)- or (1, 3)-problem to conserve its sign. In particular he shows that if

$$\alpha < \beta < \min[r_{31}(\alpha), r_{22}(\alpha)] \quad \{\alpha < \beta < \min[r_{13}(\alpha), r_{22}(\alpha)]\}$$

($r_{ij}(\alpha)$ is defined in §2), then the Green's function for the (3, 1)-problem {(1, 3)-problem} is negative in the open square $(\alpha, \beta) \times (\alpha, \beta)$. His

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