ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 3, Number 1, Winter 1973

EXISTENCE OF POSITIVE AND Φ-BOUNDED HARMONIC FUNCTIONS ON RIEMANNIAN MANIFOLDS

WELLINGTON H. OW

1. By a *Riemannian manifold* R we mean a connected, orientable C^∞-manifold of dimension $n \geqq 2$ possessing a C^∞-metric tensor. Let $\Phi(t)$ be any nonnegative real-valued function defined on $[0, \infty)$. A harmonic function u on R is said to be Φ-bounded on R if the composite function $\Phi(|u|)$ possesses a harmonic majorant on R. We denote by $H\Phi(R)$, or simply $H\Phi$, the class of all Φ-bounded harmonic functions on R and by $O_{H\Phi}$ the null class consisting of all Riemannian manifolds R on which every Φ-bounded harmonic function reduces to a constant. The problem of classifying Riemann surfaces with respect to $O_{H\Phi}$ was first attempted by Parreau [4] for the special case where Φ was increasing and convex. Later Nakai [1] completely determined $O_{H\Phi}$ for general Φ in the 2-dimensional case. Recently Ow [3] extended the Φ-bounded notion to harmonic spaces and determined $O_{H\Phi}$ there. In his paper mentioned above Nakai also considered the classification of Riemann surfaces with regular boundaries. In this investigation he partially characterized the class $SO_{H\Phi}$, where a subsurface G with regular boundary is said to belong to $SO_{H\Phi}$ if every Φ-bounded harmonic function on G which vanishes continuously on ∂G is identically zero.

The purpose of this paper is to determine the class $SO_{H\Phi}$ completely for the 2-dimensional (i.e. Riemann surface) as well as for the higher dimensional cases. An important factor in this regard is a theorem of Parreau on the existence of positive harmonic functions. In higher dimensions the class of admissible subregions will consist of smooth subregions of Riemannian n-manifolds. Here a subregion G of R will be called *smooth* if its relative boundary $\partial G \neq \emptyset$ satisfies the following: Each point $p \in \partial G$ has a neighborhood N and a diffeomorphism h of N with a region in E^n such that $h(N \cap \partial G)$ is contained in a hyperplane.

2. Before giving a characterization of $SO_{H\Phi}$ we shall first give some necessary preliminary results. The following theorem of Parreau [4],

Received by the editors June 28, 1971.
AMS (MOS) subject classifications (1970). Primary 31A05, 31B05, 30A48; Secondary 31A15, 31B15.