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ON THE REPRESENTATION OF POLYNOMIALS OVER FINITE FIELDS AS SUMS OF POWERS AND IRREDUCIBLES

WILLIAM A. WEBB

I. **Introduction.** There are a number of results known concerning the expression of an integer as the sum of a certain number of primes and kth powers [2], [3], [4]. In this paper, we prove several of these results, specifically those found in [4], for polynomial rings over finite fields.

A Hardy-Littlewood like method is used. The use of the Riemann hypothesis simplifies the proofs and enables us to obtain better error terms than those obtained in [4].

II. **Notation and preliminary results.** In general we follow the notation used in [5] and [6].

$GF[q, x]$ is the ring of polynomials over the finite field with q elements, $q = p^\beta$, p a prime.

$\mathcal{K}_{1/x}$ is the completion of the field of rational functions over $GF(q)$, with respect to ν, the degree valuation.

$\mathcal{P}_j = \{t \in \mathcal{K}_{1/x} : \nu(t) > j\}$.

$\mathcal{P}_0 = \mathcal{P}$.

$E(a) = \lambda(\alpha)$ where λ is a fixed nonprincipal character on $GF(q)$ and α is the coefficient of $1/x$ in a, where $a \in \mathcal{K}_{1/x}$.

$\int dp$ is the Haar integral on $\mathcal{P}$.

All capital letters represent elements of $GF[q, x]$.

$\deg K = \deg P_i = nk$ $(k \geqq 2)$.

$\deg A_i = n$.

P_i and A_i are primary, that is, have leading coefficient 1.

P_i are irreducible.

$\delta_i \in GF(q)$ are such that $\sum \delta_i = \operatorname{sgn} K =$ leading coefficient of K.

$\sum'$ denotes a sum over primary polynomials.

$f(t) = \sum'_{\deg P = nk} E(Pt)$.

$g(t) = \sum'_{\deg A = n} E(A^k t)$.

The main theorem we prove is

THEOREM 1. *If $p > k$, and $N_1(K)$ is the number of representations of K in the form*

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