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RADICALS OF ENDOMORPHISM NEAR-RINGS

MARJORY J. JOHNSON [1]

Several radical properties have been defined for a distributively generated (d.g.) near-ring R with identity—the radical $J(R)$, the quasi-radical $N(R)$, the ideal-radical $I(R)$, the radical-subgroup, the primitive-radical $P(R)$, and the nil-radical $L(R)$. The order of containment of the various radicals is $L(R) \subseteq I(R) \subseteq N(R) \subseteq J(R) \subseteq P(R)$ (cf. [1], [2]). The radical-subgroup is also contained in $J(R)$, but it is not known how it compares with $N(R)$ in general. If R is a ring, the radical, quasi-radical, ideal-radical, and radical-subgroup are all equal to the Jacobson radical. If R is a near-ring which is not a ring, then the above radicals are not equivalent in general, even if R is finite (cf. [2], [7]).

The purpose of this paper is to examine these radicals for the particular (left) d.g. near-ring $E(G)$, the near-ring generated by the endomorphisms of G, where G is a finite group. We show that $L(E(G)) = I(E(G)) = N(E(G)) = J(E(G)) = P(E(G))$. If G is the sum of its minimal fully invariant subgroups, then $J(E(G))$ and hence all of the radicals of $E(G)$ are $\{0\}$. If G is not the sum of its minimal fully invariant subgroups, the radical $J(E(G))$ is a proper nonzero ideal of $E(G)$. In §5 we give examples to show that in the latter situation, the radical-subgroup of $E(G)$ may or may not be equal to $J(E(G))$.

1. **Definitions.** It is assumed that the reader is familiar with the definitions of a (left) d.g. near-ring and of $E(G)$, the near-ring generated by the endomorphisms of a group G (cf. [8]). Note that all functions of G are written on the right and hence $E(G)$ is a left d.g. near-ring.

Let R be a (left) d.g. near-ring. The concepts of R-group, right module of R, ideal and right ideal are all defined in [7]. The radical properties which need to be defined for this paper are given below.

The radical $J(R)$ of R is the intersection of all annihilating ideals of the minimal R-groups (cf. [6]).

The nil-radical $L(R)$ of R is the sum of all nilpotent ideals of R (cf. [2]).

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