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DUAL AND BIDUAL LOCALLY CONVEX SPACES[1]

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ABSTRACT. A characterization of quasi-barreled spaces which are strong duals of quasi M-barreled spaces is given. Sufficient conditions are given for a quasi-barreled space E to be the strong bidual of a closed subspace F of E, when F has the inherited topology. These results are applied to spaces of continuous linear maps and to spaces of matrix maps between sequence spaces.

When is a locally convex space the strong dual (bidual) of a locally convex space? Many authors have addressed themselves to the first of these questions, see [**6**] for a partial bibliography. An answer to the second question, in the special case of Banach spaces, has been given by Shapiro, Shields, and Taylor in a paper yet unpublished. Their theorem reads:

THEOREM A. *Let F be a closed subspace of a Banach space E. The inclusion map of F into E can be extended to an isometry of F'' onto E if and only if there is a locally convex topology $\mathcal{T}$ on E satisfying*

(i) *the restriction of $\mathcal{T}$ to F is weaker than the norm topology on F,*
(ii) *the unit ball of F is $\mathcal{T}$-dense in the unit ball of E,*
(iii) *no proper closed subspace of F is $\mathcal{T}$-dense in E, and*
(iv) *the unit ball of E is $\mathcal{T}$-compact.*

In this paper we obtain results which generalize this theorem to general locally convex spaces. Our work also gives another answer to the first question listed above. In §3 we apply our results to spaces of matrix maps between sequence spaces.

We follow the terminology of Köthe [**5**] throughout. Hence if $E[\tau]$ is a locally convex space we will denote by $(E[\tau])'$ the topological dual of $E[\tau]$. Also, if a space is locally convex we mean it is also Hausdorff. We shall abbreviate locally convex space by l.c. space. We recall that $(E[\tau])'[\mathcal{T}_b(E)]$ is the strong dual of $E[\tau]$ and the strong bidual of $E[\tau]$ is the strong dual of the strong dual of $E[\tau]$. If F is a subspace of a l.c. space $E[\tau]$, the topology induced on F by τ will be denoted by τ_i. Also, if $\langle E_1, E_2\rangle$ is a dual system of vector spaces, then $\mathcal{T}_s(E_2, E_1)$, $\mathcal{T}_k(E_2, E_1)$, and $\mathcal{T}_b(E_2, E_1)$ denote the weak, Mackey, and strong topology on E_1 from E_2 respectively.

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