ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 4, Fall 1972

INCLUSION THEOREMS AND SEMICONSERVATIVE *FK* SPACES

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Dedicated to the memory of Leo Moser

1. **Introduction.** The first half of this article gives a unified inclusion theorem for FK spaces. These spaces arose in the theory of summability (see [**11**]). In that context it was natural to assume that every such space includes c, the set of convergent (complex) sequences. Throwing off the connection with matrices, the theory became more general and more satisfactory, [**2**], [**6**], with topological methods replacing classical matrix arguments. However, it now appears that these methods depend crucially on a weaker property than inclusion of c, the property which we call *semiconservative.* With this hypothesis the classical results are obtained, often in a more natural and simple way.

2. **Acknowledgement.** The results of this article were obtained in the course of three years of seminars held at Lehigh University. Many of them are due to or are inspired by Grahame Bennett and Nigel Kalton; in particular the methods of §8 were suggested by them. Theorem 7 was also obtained by W. H. Ruckle (using different methods). Part of Theorem 10 was announced in [**5**]. Professors G. W. Goes, F. P. A. Cass, and Mr. J. H. Hampson made useful suggestions.

3. ***FK* spaces.** An FK space X is a vector space of complex sequences which is also a Fréchet space (linear, complete metric) with continuous coordinates. We shall also assume that $X \supset E^\infty$ the set of all finitely nonzero sequences. A BK space is a normed FK space. An introduction to FK spaces is given in [**8**, §§11.3 and 12.4]. Our results can easily be extended to function spaces which are FH spaces [**8**, §11.3], replacing E^∞ by the set of functions with compact support or any other dense subspace.

We shall call X *o-conservative* if $X \supset c_0$, the space of null sequences, and *semiconservative* if $\sum \delta^k$ is weakly Cauchy, i.e. $\sum f(\delta^k)$ is convergent for all $f \in X'$. Here δ^k is the sequence x with $x_k = 1, x_n = 0$ for $n \neq k$. An o-conservative space is semiconservative by the general

Received by the editors October 19, 1970 and, in revised form, February 19, 1971.
AMS 1970 *subject classifications.* Primary 40C05, 40D25, 40H05, 46A45; Secondary 46A10, 46A20.